sec 4.1-4.3

① $f(x) = x^4 - 8x^2$

$f'(x) = 4x^3 - 16x$

$0 = 4x(x^2-4)$

$0 = 4x(x-2)(x+2)$

C.V. $x=0$, $x=2$, $x=-2$

② $g(t) = t\sqrt{4-t}$

$g'(t) = \sqrt{4-t} + t/2\sqrt{4-t}$

$g'(t) = \frac{2(\sqrt{4-t})^2 + t}{2\sqrt{4-t}} = \frac{8-2t+t}{2\sqrt{4-t}} = \frac{8-t}{2\sqrt{4-t}}$

C.V. $8-t=0$ $\quad 2\sqrt{4-t}=0$

$t=8$ $\quad 4-t=0$

$4=t$

③ $f(x) = xe^{-2x}$

$f'(x) = e^{-2x} + -2xe^{-2x}$

$0 = e^{-2x}(1-2x)$

$e^{-2x} \neq 0$ $\quad 1-2x=0$

$x = 1/2$

④ $f'(x) = 6x^2 - 6$ $[0,3]$

$0 = 6x^2 - 6$

$1 = x^2$

$\pm 1 = x$

x	$f(x) = 2x^3 - 6x$
0	0
1	-4
3	36

abs max: (3, 36)

abs min (1, -4)

⑤ $f(t) = x - 2\cos x$ $[0, 2\pi]$

$f'(t) = 1 + 2\sin x$

$-\frac{1}{2} = \sin x$

$7\pi/6, 11\pi/6 = x$

x	$f(x)$
0	-2
$7\pi/6$	~5.397
$11\pi/6$	4.027
2π	$2\pi - 1 \approx 5.283$

abs min (0, -2)

abs max ($7\pi/6$, 5.397)

⑥ $y = x\ln(x+3)$ $[0,3]$

$y' = \ln(x+3) + \frac{x}{x+3}$

$x = -1.145$

(w/ tech

x	$f(x)$
0	0
3	5.375

abs min (0, 0)

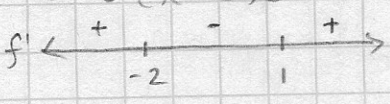
abs max (3, 5.375)

⑦ $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$0 = 6(x^2 + x - 2)$

$0 = 6(x+2)(x-1)$



inc: $(-\infty, -2)$ $(1, \infty)$

dec: $(-2, 1)$

min at $x = 1$

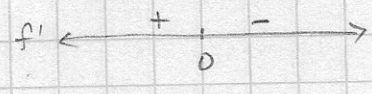
max at $x = -2$

⑧ $f(x) = (1-x)e^x$

$f'(x) = -1e^x + (1-x)e^x$

$f'(x) = e^x(-1+1-x)$

$0 = e^x(-x) \rightarrow x = 0$



inc $(-\infty, 0)$

dec $(0, \infty)$

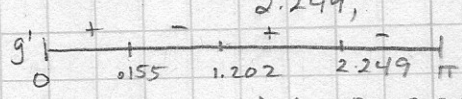
max at $x = 0$

⑨ $g(x) = 2\sin 3x + 4\cos 3x$ $[0, \pi]$

$g'(x) = 6\cos 3x - 12\sin 3x$

$0 = 6\cos 3x - 12\sin 3x$

(w/ calc) $x = 0.155, 1.202, 2.249,$



inc: $[0, .155)$ $(1.202, 2.249)$

dec: $(.155, 1.202)$ $(2.249, \pi]$

min at $x = 1.202$

max at $x = .155, 2.249$

Sec 4.1-4.3 (con't)

(10) $f(x) = 3x\sqrt{4-x^2}$ D: $-2 < x < 2$

$f'(x) = 3\sqrt{4-x^2} + \frac{3x \cdot -2x}{2\sqrt{4-x^2}}$

$f'(x) = \frac{3(\sqrt{4-x^2})^2}{(\sqrt{4-x^2})} - \frac{3x^2}{\sqrt{4-x^2}}$

$f'(x) = \frac{12-6x^2}{\sqrt{4-x^2}}$

$f''(x) = \frac{\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}(-12x) - (12-6x^2)\left(\frac{-2x}{2\sqrt{4-x^2}}\right)}{4-x^2}$

$f''(x) = \frac{-48x + 12x^3 + 12x - 6x^3}{(\sqrt{4-x^2})(4-x^2)}$

$f''(x) = \frac{6x^3 - 36x}{(4-x^2)^{3/2}}$

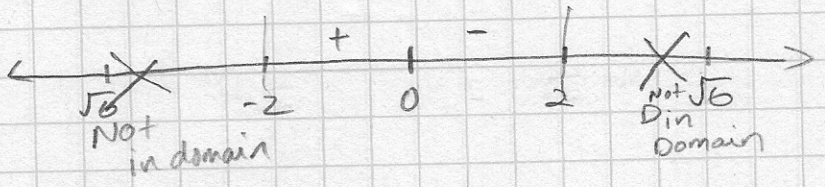
$6x^3 - 36x = 0$

$6x(x^2-6) = 0$

$x = 0 \quad x = \pm\sqrt{6}$

$4 - x^2 = 0$

$x = \pm 2$



Concave up
$(-2, 0)$

concave down
$(0, 2)$

(11) $g(x) = x^2 e^{-x}$

$g'(x) = 2xe^{-x} + x^2 e^{-x}(-1)$

$g'(x) = e^{-x}(2x - x^2)$

$g''(x) = -e^{-x}(2x - x^2) + e^{-x}(2 - 2x)$

$g''(x) = e^{-x}(-2x + x^2 + 2 - 2x)$

$g''(x) = e^{-x}(x^2 - 4x + 2)$

$e^{-x} \neq 0 \quad x^2 - 4x + 2 = 0$

w/calc. $x = .586$

$x = 3.414$

concave up: $(-\infty, .586)$ $(3.414, \infty)$

concave down: $(.586, 3.414)$

(12) (f')

inc: $(-4,-2)(4,8)(8,10)$

dec: $(-2,0)(0,4)$

min: $x = 4$

max: $x = -2$

(f'')

concave up: $(-1,0)(3,6)(8,10)$

concave down: $(-4,-2)(0,3)(6,8)$

pt of Inf. $x = -1, 0, 3, 6, 8$

(13) (f')

incr: $(5,8)$

decr: $(-\infty,-1)(-1,2)(2,5)$

min: $x = 5$

max: $\varnothing$

(f'')

concave ↑: $(-1,2)$

concave ↓: $(-\infty,-1)(2,5)(5,8)$

pt of inf: $x = 2$

(not at $x = -1$ since $f(-1)$ is undef.)

4.4

① $C(x) = 10000 + 90x - .05x^2$

$C'(x) = -90 + .1x$

$0 = -90 + .1x$

$.1x = 90$

$x = 900$

− | + (900)

at $x = 900$

② $x + 3y = S$

$xy = 192 \rightarrow x = 192/y$

$\frac{192}{y} + 3y = S$

$192y^{-1} + 3y = S$

$-192y^{-2} + 3 = S'$

$3 = \frac{192}{y^2}$

$\frac{192}{3} = y^2$

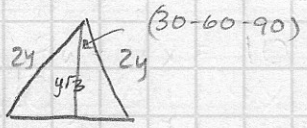
$y = \pm\sqrt{192/3}$

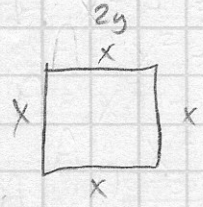
$y = \sqrt{192/3}$

$x = 192/\sqrt{192/3}$

$x = \sqrt{3 \cdot 192}$

③ (30-60-90)





$6y + 4x = 10 \rightarrow 4x = 10 - 6y$

$x = \frac{10}{4} - \frac{6}{4}y$

$A = x^2 + \frac{1}{2}(2y)(\sqrt{3}y)$

$A = x^2 + y^2\sqrt{3}$

$A = (5/2 - 3/2y)^2 + y^2\sqrt{3}$

$A' = 2(5/2 - 3/2y)^1 \cdot (-3/2) + 2y\sqrt{3}$

$0 = -3(5/2 - 3/2y) + 2y\sqrt{3}$

$0 = -15/2 + 9/2y + 2y\sqrt{3}$

w/ calc.

$y = .942$

$2y = 1.884$ (each side of triangle)

$x = 10/4 - 6/4(.942)$

$x = 1.087$ (each side of square)

④ Same as #3

4.5 (only 1-3)(skip 4-6)

① $f(x) = x^3 - 2x + 3$ $\quad f'(x) = 3x^2 - 2$

$f(2) = 2^3 - 2(2) + 3$ $\quad f'(2) = 3(2)^2 - 2$

$f(2) = 7$ $\quad f'(2) = 10$

$y = 10(x-2) + 7$

$f(2.1) \approx 10(2.1-2) + 7$

$\approx 10(.1) + 7$

≈ 8

② $f(x) = \sqrt{x^2+9}$ $\quad f'(x) = \frac{2x}{2\sqrt{x^2+9}}$

$f(-4) = \sqrt{16+9}$ $\quad f'(-4) = \frac{-4}{\sqrt{25}} = -\frac{4}{5}$

$f(-4) = 5$

$y = -4/5(x+4) + 5$

$f(-3.99) \approx -4/5(-3.99+4) + 5$

$\approx -4/5(.01) + 5$

≈ 5.008

③ $f(x) = \tan x$ $\quad f'(x) = \sec^2 x$

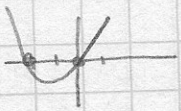
$f(\pi) = \tan \pi$ $\quad f'(\pi) = (-1)^2$

$= 0$ $\quad = 1$

$y = 1(x - \pi) + 0$

$f(3.2) \approx 1(3.2 - \pi) \approx .0584$

4.6



[-7, 2]

① $f(x) = 5 - \frac{4}{x}$ $\quad$ Not cont $x=0$ but not interval

[1, 4]

$f(1) = 5 - 4 = 1$

$f(4) = 5 - 1 = 4$

$m = \frac{4-1}{4-1} = \frac{3}{3} = 1$

$f'(x) = +4/x^2$

$1 = \frac{4}{x^2}$

$x^2 = 4$

$x = \pm 2$

↑ only 2 is in interval

$x = 2$

② $f(x) = \sqrt{x(x+2)}$

Domain: $x \geq 0$ $x \leq -2$

hypoth. of MVT not satisfied since $f(x)$ DNE between -2 and 0 and these values are in the interval

③ [-1, 3]

$f(x) = \frac{x^2-1}{x-2}$

Domain: $x \neq 2$

hyp. of MVT is not satisfied since $x=2$ is in the interval

④ $f(x) = x^{2/3}$

continuous everywhere

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

Not Diff at $x=0$

Not Diff at $x=0$, can be an endpt $[0,8]$ but can't be an interior pt. $[-1,0]$

⑤ temp change over time is cont/diff.

(0, 1550)

(5, 390)

$m = \frac{390-1500}{5-0} = -222°/hr$

The MVT guarantees a "c" for

$f'(c) = -222°/hr$

4.7

① $\lim_{x\to 2} \frac{x^3-x-2}{x-2} = \frac{4}{0}$ can't use

② $\lim_{x\to\infty} \frac{(\ln x)^3}{x} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{3(\ln x)^2 \cdot \frac{1}{x}}{1} = \lim_{x\to\infty} \frac{3(\ln x)^2}{x} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{6\ln x \cdot \frac{1}{x}}{x} = \lim_{x\to\infty} \frac{6\ln x}{x} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{6/x}{1} = \frac{6}{\infty} = 0$

③ $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x} = \frac{0}{0}$ ✓

$\lim_{x\to 0} \frac{\frac{1}{2\sqrt{4-x^2}} \cdot -2x}{1}$

$\lim_{x\to 0} \frac{-x}{\sqrt{4-x^2}} = \frac{0}{0}$ ✓

$\lim_{x\to 0} \frac{-1}{-2x/\sqrt{4-x^2}} = \lim_{x\to 0} \frac{\sqrt{4-x^2}}{x} = \frac{0}{0}$ ✓

EEK vicious circle!

find another way

④ $\lim_{x\to 0} \frac{e^x-(1-x)}{x} = \frac{0}{0}$ ✓

$\lim_{x\to 0} \frac{e^x+1}{1} = 2$

⑤ $\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)} \quad \frac{0}{0}$ ✓

$\lim_{x\to 0} \frac{2\cos(2x)}{3\cos(2x)} = \frac{2}{3}$

⑥ $\lim_{x\to\infty} \frac{3x^2-2x+1}{2x^2+3} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{6x-2}{4x} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{6}{4} = \frac{3}{2}$

⑦ $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{1}{2x/2\sqrt{x^2+1}}$

$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x} = \frac{\infty}{\infty}$ ✓

another vicious cycle EEK

$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}} = 1$

⑧ $\lim_{x\to\infty} \frac{x^2}{e^{5x}} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{2x}{5e^{5x}} = \frac{\infty}{\infty}$ ✓

$\lim_{x\to\infty} \frac{2}{25e^{5x}} = \frac{2}{\infty} = 0$