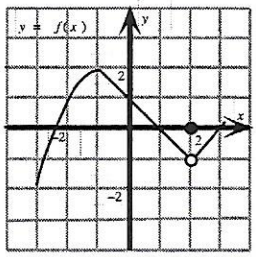
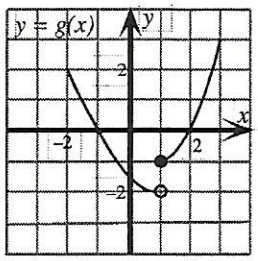
Review for Test, Limits and Continuity

On the test, you will be required to show all of your work. Completion of this worksheet is optional, and it will not be graded. However, the problems are excellent practice for the test, and you are encouraged to ask about any questions that you do not understand. The test will count 100 points,

Use the graphs to find each limit, if it exists.





1. $\lim_{x\to 2}\big(f(x)+g(x)\big)$
2. $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$
3. $\lim_{x\to 1}\sqrt{3+f(x)}$
4. $\lim_{x\to 2}x^3 f(x)$
5. $\lim_{x\to 1^-}g(x)$
6. $\lim_{x\to 2^+}\dfrac{f(x)}{g(x)}$
7. $\lim_{x\to 2}f\big(g(x)\big)$
8. $\lim_{x\to -1}\big(2f(x)g(x)\big)$

9. Find the value(s) of x for which $h(x)=\dfrac{2x+1}{2x^2+11x+5}$ is discontinuous and identify these discontinuities as removable or nonremovable.

10. Draw a function with each of the following characteristics:
    a) point discontinuity
    b) infinite discontinuity
    c) jump discontinuity
    d) oscillating discontinuity

11. Write the definition of continuity at a point.

Find each limit, if it exists.

13. $\lim_{x\to 2}\sec\dfrac{\pi x}{3}$
14. $\lim_{x\to 0}\dfrac{x+\sin x}{x}$
15. $\lim_{x\to 3^-}\sqrt{9-x^2}$
16. $\lim_{x\to\infty}\dfrac{2x}{\sqrt{x^2+1}}$
17. $\lim_{x\to 2}\dfrac{x^3-1}{x-2}$
18. $\lim_{x\to 2}\dfrac{x^2-4}{x^2+4}$
19. $\lim_{h\to 0}\dfrac{(3+h)^2-9}{h}$
20. $\lim_{x\to 1}\left(5-\dfrac{2}{(x-1)^2}\right)$
21. $\lim_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$
22. $\lim_{x\to 0}f(x)$, if $f(x)=\begin{cases}\dfrac{|x|}{x}, & x\neq 0\\ 0, & x=0\end{cases}$
23. $\lim_{x\to -2}(x-6)^{2/3}$
25. $\lim_{x\to -\infty}\dfrac{5x^3-2x+7}{3x^3+2x^2}$

For the next three questions, use $f(x)=\begin{cases}3-x, & x<2\\ \dfrac{x}{2}+1, & x>2\end{cases}$.

26. $\lim_{x\to 2^+}f(x)$
27. $\lim_{x\to 2^-}f(x)$
28. $\lim_{x\to 2}f(x)$

For the next three questions, use $f(x)=\begin{cases}3-x, & x<2\\ 2, & x=2.\\ \frac{x}{2}, & x>2\end{cases}$

29. $\lim_{x\to 2^+} f(x)$
30. $\lim_{x\to 2^-} f(x)$
31. $\lim_{x\to 2} f(x)$

For the next three questions, let $f(x)=\dfrac{x^2-3x-10}{x^2-4}$.

32. Find $\lim_{x\to -2} f(x)$, if it exists.
33. Sketch a graph of $f(x)$.
34. Find a new function, $g(x)$, that agrees with $f(x)$ at all but one point and is defined at $x=-2$.

35. Determine the value of c such that each function is continuous on $(-\infty, \infty)$.

a) $f(x)=\begin{cases}3x-2, & x<5\\ x^2+c, & x\geq 5\end{cases}$ b) $g(x)=\begin{cases}x^2-1, & x<3\\ 2cx, & x\geq 3\end{cases}$

36. Find all vertical and horizontal asymptotes of each function.

a) $g(x)=\dfrac{x-2}{2x^2+3x-5}$ b) $f(x)=\dfrac{x^2-2x}{x+2}$ c) $h(x)=\dfrac{x^2-4}{x^2+4}$

37. In your own words, explain the Intermediate Value Theorem. Include a sketch with your answer.

38. Identify all asymptotes and discontinuities of the graph of $y=\dfrac{3x^2-x+5}{x^2-4}$.

For the next three questions, let $f(x)=\begin{cases}\dfrac{x^2+x}{x}, & x\neq 0\\ 0, & x=0\end{cases}$.

True or false. Justify your answers.

39. $\lim_{x\to 0} f(x)$ exists.
40. $f(0)$ exists.
41. f is continuous at $x=0$.

42. Let $g(x)=4x^2-x$.
    a) Find the average rate of change of g on $[0, 5]$.
    ~~b) Find the rate of change of g at $x=2$.~~
    ~~c) Find the equation of the tangent line at $x=2$.~~
    ~~d) Find the equation of the normal line at $x=2$.~~

Study Guide Solutions

① $\lim_{x\to 2}(f(x)+g(x))$

$\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$= -1 + 0$

$= -1$

② $\lim_{x\to -1} \frac{f(x)}{g(x)}$

$= \frac{2}{0}$

= undef.

③ $\lim_{x\to 1} \sqrt{3+f(x)}$

$= \sqrt{3+0}$

$= \sqrt{3}$

④ $\lim_{x\to 2} x^3 \cdot f(x)$ (#5)

$= (2)^3(-1)$

$= 8(-1)$

$= -8$

⑥ $\lim_{x\to 2^+} \frac{f(x)}{g(x)}$

$\frac{-1}{-1}$

$= 1$

⑦ $\lim_{x\to 2} f(g(x))$

$= f(0)$

$= 1$

⑤ $\lim_{x\to 1^-} g(x)$

$= -2$

⑧ $\lim_{x\to -1} (2f(x)g(x)$

$= 2(2)(0)$

$= 0$

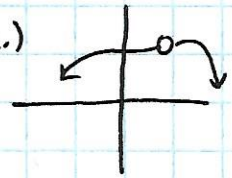
⑨ $h(x) = \frac{2x+1}{(2x^2+11x+5)}$

$h(x) = \frac{2x+1}{(2x+1)(x+5)}$

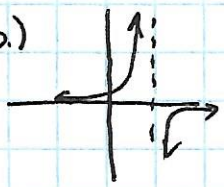
Discont: $x = -\frac{1}{2}$ removeable (hole)

$x = -5$ nonremovable (asymptote)

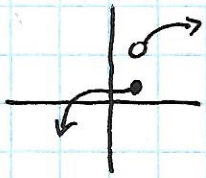
⑩ a.)



b.)



c.)



d.

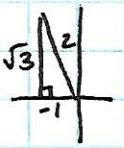
⑪ if $f(a) = \lim_{x\to a} f(x)$

the function is continuous at point a.

⑫ Skip

⑬ $\lim_{x\to 2} \sec\frac{\pi x}{3}$

$= \sec\frac{2\pi}{3}$



$= -\frac{2}{1}$ or -2

⑭ Skip

⑮ $\lim_{x\to 3^-} \sqrt{9-x^2}$

$= \sqrt{9-3^2}$

$= \sqrt{0}$

$= 0$

⑯ $\lim_{x\to -\infty} \frac{2x}{\sqrt{x^2+1}}$

$\lim_{x\to -\infty} \frac{2x}{|x|}$

$= 2$

⑰ $\lim_{x\to 2} \frac{x^3-1}{x-2}$ = DNE

$\lim_{x\to 2^-} \frac{x^3-1}{x-2} = \frac{+}{-} = -\infty$ $\lim_{x\to 2^+} \frac{x^3-1}{x-2} = \frac{+}{+} = \infty$

(18) $\lim_{x\to 2} \frac{x^2-4}{x^2+4}$

$= \frac{4-4}{4+4}$

$= 0$

(19) $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$

$\lim_{h\to 0} \frac{((3+h)-3)((3+h)+3)}{h}$

$\lim_{h\to 0} \frac{h(6+h)}{h}$

$= 6+0$

$= 6$

(20) $\lim_{x\to 1} \left(5 - \frac{2}{(1-x)^2}\right)$

$\lim_{x\to 1} 5 - \lim_{x\to 1} \frac{2}{(1-x)^2}$

$= 5 - \infty$

$= -\infty$

(21) ~~$\lim_{x\to 0} \frac{\frac{2}{2(2+x)} - \frac{1}{2}\frac{(2+x)}{(2+x)}}{x}$~~

~~$\lim_{x\to 0} \frac{\frac{2-2-x}{2(2+x)}}{x}$~~

~~$\lim_{x\to 0} \frac{-x}{2(2+x)} \cdot \frac{1}{x}$~~

~~$\lim_{x\to 0} \frac{-1}{2(2+x)}$~~

Try Again ☺

(21) $\lim_{x\to 0} \frac{\frac{2\cdot 1}{2(2+x)} - \frac{1}{2}\frac{(2+x)}{(2+x)}}{x}$

$\lim_{x\to 0} \frac{\frac{2-2-x}{2(2+x)}}{x}$

$\lim_{x\to 0} \frac{-x}{2(2+x)} \cdot \frac{1}{x}$

$= \frac{-1}{2(2+0)} = -\frac{1}{4}$

(22) $\lim_{x\to 0} f(x)$ $\quad f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$\lim_{x\to 0^-} f(x) = \frac{+x}{-x} = -1$

$\lim_{x\to 0^+} f(x) = \frac{+x}{+x} = 1$

since $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$ the limit DNE

(23) $\lim_{x\to -2} (x-6)^{2/3}$

$= (-2-6)^{2/3}$

$= (-8)^{2/3}$

$= 64^{1/3}$

$= 4$

(25) $\lim_{x\to -\infty} \frac{5x^3-2x+7}{3x^3+2x^2}$

$= \frac{5}{3}$

(26) $\lim_{x\to 2^+} f(x)$

$= \frac{2}{2} + 1$

$= 2$

(27) $\lim_{x\to 2^-} f(x)$

$= 3 - 2$

$= 1$

(28) $\lim_{x\to 2} f(x)$

DNE

(29) $\lim_{x\to 2^+} f(x)$

$= \frac{2}{2}$

$= 1$

(30) $\lim_{x\to 2^-} f(x)$

$= 3 - 2$

$= 1$

(31) $\lim_{x\to 2} f(x)$

$= 1$

$$f(x) = \frac{x^2-3x-10}{x^2-4} = \frac{(x-5)(x+2)}{(x-2)(x+2)}$$ hole; $x-2$: asym

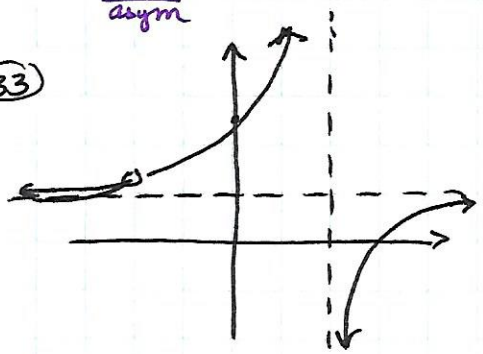
(32) $\lim_{x\to -2} \frac{(x-5)(x+2)}{(x-2)(x+2)}$

$= \frac{-2-5}{-2-4}$

$= -7/-6$

$= \frac{7}{6}$

(33)



(34) $f(x) = \frac{x-5}{x-2}$

(35) $f(x) = \begin{cases} 3x-2 & x<5 \\ x^2+c & x \geq 5 \end{cases}$

$3(5)-2 = (5)^2+c$

$15-2 = 25+c$

$13 = 25+c$

$-12 = c$

(36) $g(x) = \begin{cases} x^2-1 & x<3 \\ 2cx & x \geq 3 \end{cases}$

$(3)^2-1 = 2c(3)$

$9-1 = 6c$

$8 = 6c$

$\frac{8}{6} = c$

(37) See Notes from 8/24

(38) $y = \frac{3x^2-x+5}{x^2-4} = \frac{(3x \quad x \quad)}{(x-2)(x+2)}$ Doesn't factor

asymptotes $x = 2$ $x = -2$ (vertical)

$y = 3$ (horizontal)

(39) $\lim_{x\to 0^+} \frac{x^2+x}{x} = \frac{+}{+} = +\infty$

$\lim_{x\to 0^-} \frac{x^2+x}{x} \quad \frac{+}{-} = -\infty$

limit does not exist

False

(40) $f(0)$ exists

True

$f(x) = 0$ if $x=0$

(41) $f(x)$ is continuous at $x=0$

False

$f(0) \neq \lim_{x\to 0} f(x)$

(42) $g(x) = 4x^2 - x$ average rate on $[0,5]$

$(0,0)$ $y = 4(0)^2 - 0 = 0$

$(5,95)$ $y = 4(5)^2 - 5$

$m = \frac{95}{5} = 19$