Unit 7 Study Guide

① $y = Ce^{-3t}$

$y' = -3Ce^{-3t}$

verify: $y'(t) + 3y = 0$

$(-3Ce^{-3t}) + 3(Ce^{-3t}) = 0$

$0 = 0$ ✓

Yes

② $y = C_1 \sin 2t + C_2 \cos 2t$

$y' = 2C_1 \cos 2t - 2C_2 \sin 2t$

$y'' = -4C_1 \sin 2t - 4C_2 \cos 2t$

verify: $y''(t) + 4y = 0$

$(-4C_1 \sin 2t - 4C_2 \cos 2t) + 4(C_1 \sin 2t + C_2 \cos 2t) = 0$

$-4C_1 \sin 2t - 4C_2 \cos 2t + 4C_1 \sin 2t + 4C_2 \cos 2t = 0$

$0 = 0$ ✓

Yes

③ $\int u'' = \int 8e^{2x} - 8e^{-2x}$; $u(0) = 9$, $u'(0) = 15$

$u' = 4e^{2x} + 4e^{-2x} + C$

$15 = 4e^0 + 4e^0 + C$

$15 = 8 + C$

$7 = C$

$\int u' = \int 4e^{2x} + 4e^{-2x} + 7$

$u = 2e^{2x} - 2e^{-2x} + 7x + C$

$9 = 2e^0 - 2e^0 + 7(0) + C$

$9 = 0 + C$

$9 = C$

$u(x) = 2e^{2x} - 2e^{-2x} + 7x + 9$

④ $x^8 \frac{dy}{dx} = y^2$; $x > 0$

$\frac{1}{y^2} dy = \frac{1}{x^8} dx$

$y^{-2} dy = x^{-8} dx$

$\frac{y^{-1}}{-1} = \frac{x^{-7}}{-7} + C$

$\frac{1}{y} = \frac{1}{7x^7} + \frac{7C_1 x^7}{7x^7}$

$\frac{1}{y} = \frac{1 + Cx^7}{7x^7}$

$y = \frac{7x^7}{1 + Cx^7}$

⑤ $3y^2 \frac{dy}{dx} - 3x = 0$

$3y^2 \frac{dy}{dx} = 3x$

$3y^2 dy = 3x\,dx$

$\frac{3y^3}{3} = \frac{3x^2}{2} + C$

$y^3 = \frac{3x^2}{2} + C$

$y = \sqrt[3]{\frac{3}{2}x^2 + C}$

⑥ $u'(x) = e^{9x - 4u}$

$\frac{du}{dx} = \frac{e^{9x}}{e^{4u}}$

$e^{4u} du = e^{9x} dx$

$\frac{1}{4} e^{4u} = \frac{1}{9} e^{9x} + C$

$\ln e^{4u} = \ln\left(\frac{4}{9} e^{9x} + C\right)$

$4u = \ln\left(\frac{4}{9} e^{9x} + C\right)$

$u = \frac{1}{4}\left(\ln\left(\frac{4}{9} e^{9x} + C\right)\right)$

⑦ $y'(t) = y(4t^3 + 5t^4)$; $y(0) = 6$

$\frac{1}{y} \cdot \frac{dy}{dt} = 4t^3 + 5t^4$

$\frac{1}{y} dy = (4t^3 + 5t^4) dt$

$\ln|y| = t^4 + t^5 + C$

$\ln|6| = 0 + 0 + C$

$\ln 6 = C$

$e^{\ln|y|} = e^{t^4 + t^5 + \ln 6}$

$y = e^{t^4 + t^5} \cdot e^{\ln 6}$

$y = 6e^{t^4 + t^5}$

(8) $y = Ce^{-2t}$

$y' = -2Ce^{-2t}$

verify: $y'(t) + 2y = 0$

$(-2Ce^{-2t}) + 2(Ce^{-2t}) = 0$

$-2Ce^{-2t} + 2Ce^{-2t} = 0$

$0 = 0$ ✓

Yes

(9) $y'(x) = \frac{6-x}{y+11}$ $y(-5) = -5$

$\frac{dy}{dx} = \frac{6-x}{y+11}$

$\int (y+11)\,dy = \int (6-x)\,dx$

$\frac{y^2}{2} + 11y = 6x - \frac{x^2}{2} + C$

$\frac{(-5)^2}{2} + 11(-5) = 6(-5) - \frac{(-5)^2}{2} + C$

$25 + -55 = -30 + C$

$0 = C$

$\frac{y^2}{2} + 11y = 6x - \frac{x^2}{2}$

or

$y^2 + 22y = 12x - x^2$

(10) $\frac{dy}{dt} = -Ky^n$ $(n=3)$ $y(0)=3$ $k=.9$

$\frac{1}{y^3}\,dy = -K\,dt$

$y^{-3}\,dy = -.9\,dt$

$\frac{y^{-2}}{-2} = -.9t + C$

$y^{-2} = -1.8t + C$

$\frac{1}{9} = -1.8(0) + C$

$\frac{1}{9} = C$

$y^{-2} = -1.8t + \frac{1}{9}$

$\frac{1}{y^2} = \frac{-1.8t \cdot 9 + 1}{9}$

$\frac{1}{y^2} = \frac{-16.2t + 1}{9}$

$y^2 = \frac{9}{-16.2t+1}$

$y = +\sqrt{\frac{9}{-16.2t+1}}$

$y = \frac{3}{\sqrt{-16.2t+1}}$

(11) ~~$y'(t) = .5y(1-y)$~~

SKIP → Logistic

(12) • decreases 1% every 500 yrs → $y(500) = .99$

• started with 100% → $y(0) = 1$

$y = y_0 e^{kt}$

$y = 1e^{kt}$

$.99 = e^{k(500)}$

$\ln(.99) = k(500)$

$\frac{\ln .99}{500} = k$

$-.00002 = k$

(13) (like number 12)

• decreases 9% per year → $y(1) = .91$

• Started with 100% $y(0) = 1$

$y = y_0 e^{kt}$

$y = 1e^{kt}$ ← Solve for k w/ $y(1) = .91$

$.91 = e^{k(1)}$

$\ln(.91) = k$

$-.0943 \approx k$

$y = e^{-.0943t}$

Need to find out time to get to 1/8 of original

$\frac{1}{8} = e^{-.0943t}$

$\frac{\ln(1/8)}{-.0943} = t$

$22.04 \approx t$

↓ round up time since it hits 1/8 in it's 22nd yr.

$t = 23$ yrs